Review
of
Zuse, Horst: Software Complexity – Measures and Methods. / Berlin, New York, NY: de Gruyter (1991), 605 pp. ISBN 3-11-012226-X

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The aim of this book, as outlined by the author in the preface, is *"to prepare the reader for a detailed study of the methods of application of measurement theory, the definition and use of scales, the description of measures as ordinal or ratio scale, ... and the application of software complexity measurement in practice. The book presents ... a theoretical foundation of the measurement of software complexity, and ... a detailed discussion of more than ninety software complexity measures and their application to software complexity measurement."*

Unfortunately, the reader who approaches this book with high expectations will be disappointed. A careful study reveals grave shortcomings: Measurement theory is misrepresented; the theoretical foundations of software complexity measurement are unsound; the methods for the practical application of software complexity measurement are insufficiently elaborated. The best features of the book are the nearly complete collection of complexity measures and the extensive annotated bibliography.

After a brief discussion of software measurement the author introduces measurement theory (ch. 4). He refers primarily to the standard texts of Krantz et al. (*Foundations of Measurement*, Academic Press, 1971) and Roberts (*Measurement Theory*, Addison-Wesley, 1979). By combining axioms taken from both texts, he obtains an incorrect version of the representation theorem for an extensive structure (cf. Krantz et al. pp. 73-74, Roberts pp. 127-128). The same lack of mathematical sophistication is apparent in the author's definition of a modified extensive structure for flowgraphs (p. 57):

Let $\mathbf{P}$ be a nonempty set of flowgraphs, $\bullet \geq$ a binary relation on $\mathbf{P}$, and $\circ$ a binary operation on $\mathbf{P}$. The relational system $(\mathbf{P}, \bullet \geq, \circ)$ is a modified extensive structure if and only if the following axioms are satisfied for all flowgraphs $P_1, ..., P_4$:

A_1: a) For all P_1, P_2, P_3: If $P_1 \bullet \geq P_2$ and $P_2 \bullet \geq P_3$, then $P_1 \bullet \geq P_3$ (transitivity).

b) For all P_1, P_2: $P_1 \bullet \geq P_2$ or $P_2 \bullet \geq P_1$ (completeness). (I. e. $(\mathbf{P}, \bullet \geq, \circ)$ is a weak order.)

A_2: For all P_1, P_2, P_3: $(P_1 \circ P_2) \circ P_3 \approx P_1 \circ (P_2 \circ P_3)$ (weak associativity).

A_3: For all P_1, P_2: $P_1 \circ P_2 \approx P_2 \circ P_1$ (weak commutativity).

A_4: For all P_1, P_2, P_3: If $P_1 \bullet \geq P_2$, then $P_1 \circ P_3 \bullet \geq P_1 \circ P_3$ (weak monotonicity).

A_5: For all P_1, P_2, P_3, P_4: If $P_3 \bullet > P_4$, then there is an integer n such that $P_1 \circ nP_3 \bullet > P_2 \circ nP_4$ (Archimedean property).

A flowgraph P is a directed graph that has unique entry and exit nodes, and every node of P lies on some path from the entry node to the exit node. The intended meaning of the relation $\bullet \geq$ is "more complex than or as complex as". The operation $\circ$ is interpreted as sequential concatenation (BSEQ) and as alternating concatenation (BALT). The relations $\bullet >$ and $\approx$ are not explained, but can be defined as follows: $P \bullet > P'$ iff $P \bullet \geq P'$ and not $(P' \bullet \geq P)$; $P \approx P'$ iff $P \bullet \geq P'$ and $P' \bullet \geq P$. It is important to note that the definition of the relation $\approx$ is critical to the representation problem. In order to obtain necessary and sufficient conditions for extensive measurement, which is the author's objective, the relation $\approx$ has to be defined differently (Roberts p. 126): $P \approx P'$ iff not $(P \bullet \geq P')$ and not $(P' \bullet \geq P)$.

The author's definition of the modified extensive structure is questionable. Since he does not deal with the representation problem, it is not clear whether the modified extensive structure is in fact a measurement structure. There is strong evidence indicating that this is not the case (cf. Roberts pp. 126-129). A negative solution of the representation problem implies that there is no ratio scale, i. e. a homomorphism h from the relational system $(\mathbf{P}, \bullet \geq, \circ)$ to the real numbers where h is unique up to multiplication by a positive constant.

This fatal consequence is surprisingly irrelevant to the author's main argument. For he actually uses a different "axiom system" which is hidden in a computer program called MDS (measure demonstration system). The following pseudo-axioms have been reconstructed from the context of the book:

A_1': a) For all P_1, P_2, P_3: If $\mu(P_1) \geq \mu(P_2)$ and $\mu(P_2) \geq \mu(P_3)$, then $\mu(P_1) \geq \mu(P_3)$.

b) For all P_1, P_2: $\mu(P_1) \geq \mu(P_2)$ or $\mu(P_2) \geq \mu(P_1)$.

A_2': For all P_1, P_2, P_3: $\mu((P_1 \circ P_2) \circ P_3) = \mu(P_1 \circ (P_2 \circ P_3))$.

A_3': For all P_1, P_2: $\mu(P_1 \circ P_2) = \mu(P_2 \circ P_1)$.

A_4': For all P_1, P_2, P_3: If $\mu(P_1) \geq \mu(P_2)$, then $\mu(P_1 \circ P_3) \geq \mu(P_2 \circ P_3)$.

A_5': For all P_1, P_2, P_3, P_4: If $\mu(P_3) > \mu(P_4)$, then there is an integer n such that $\mu(P_1 \circ nP_3) > \mu(P_2 \circ nP_4)$.

A complexity measure μ is a real-valued function on the set of flowgraphs $\mathbf{P}$. The author investigates 98 complexity measures. He asks the question: Which of the pseudo-axioms A_1',..., A_5' are true for a specific measure μ? In order to answer this question he uses the program MDS for ca. 200 test flowgraphs. If all five pseudo-axioms are satisfied in the domain of test flowgraphs for some measure μ, then the author claims that "the measure μ is an extensive structure" or "the measure μ is an extensive structure and a ratio scale".

The author's approach gives rise to the following criticisms:

1. The axioms A_1,..., A_5 are logically independent from the pseudo-axioms A_1',..., A_5'. This means that all axioms can be false, although all pseudo-axioms are true.

2. In order to prove that a pseudo-axiom is true for some measure μ, it is not sufficient to consider 200 test flowgraphs. Yet a pair, a triple or a quadruple of flowgraphs is sufficient to disprove a pseudo-axiom for some μ.

3. The author does not deal with transitivity, completeness, weak associativity, weak monotonicity, nor the Archimedean property with respect to the relational system $(\mathbf{P}, \bullet \geq, \circ)$ Instead he examines whether measures μ are compatible with his two definitions of a concatenation operation $\circ$.

4. The pseudo-axiom A_1' is true for all real-valued measures μ. This is implied by the transitivity and completeness of the order relation on the set of real numbers. Using the author's fallacy, i. e. transfering results about pseudo-axioms to axioms, it follows that all real-valued measures on flowgraphs are ordinal scales (which is obviously not true).

5. The investigation on conditions for ordinal scales - an essential part of the book - is deficient in other respects. The author establishes 21 theorems of the type "a measure μ is completely described as an ordinal scale" (cf. overview of results, pp. 531-532), but he does not supply a complete proof for any of these "theorems".

The main conclusions of the book are compiled in Figure 9.17 (pp. 531-532). The table is misleading, in that the entries for ordinal scales are unsubstantiated and the entries for axioms actually refer to pseudo-axioms. Therefore the results of the form "measure μ is completely described as an ordinal scale" are merely conjectures and the results of the form "measure μ is an extensive structure" or "measure μ is an extensive structure and a ratio scale" are deriven by manipulating terminology.

To summarize: The bridge between measurement theory and software complexity is poorly constructed; mathematical treatment is lacking in thouroughness; fundamental concepts are distorted and misapplied; theorems are presented without proofs. The author has failed to achieve his goal of writing a useful book – pseudo-mathematics of software complexity benefits no one.

Postscript 2003: The arguments of the review are still valid. Zuse did not succeed in showing that the "modified extensive structure" is a measurement structure. In a second book on software measurement he presents a bogus proof (*A Framework of Software Measurement*, de Gruyter, 1998, pp. 673-676).